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Dynamic Bulk Moduli of Several Solids Impacted by Weak Shockwaves

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The bulk compressibility of several solids was measured using attenuated underwater shockwaves. Attenuation was such as to deliver an approximate sonic pulse to the test specimens. Measurements were made of the wave-attenuation and the wave-transit time in several metals, plastics, and chemical compounds. High-speed smear-camera shadowgraphs were used for the measurements.

INTRODUCTION

For a shockwave moving through material initially in a state with pressure P_0, specific volume V_0, and specific internal energy E_0, the equations conserving mass, momentum, and energy are

$$P-P_0=(1/V_0)U_s u_p, \tag{1}$$

$$V_0/V=U_s/(U_s-u_p), \tag{2}$$

and

$$E-E_0=\tfrac{1}{2}(P+P_0)(V_0-V), \tag{3}$$

where P, E, and V are the pressure, energy, and specific volume; U_s is the velocity of the shockwave; and u_p is the particle velocity of the material behind the shock front. An analysis of the P, V, and E data in the compressed state is provided by the solution of Eq. 3, which gives as a projection in the P–V plane the so-called Hugoniot curve. Experimentally, however, one usually measures the parameters in the U_s–u_p plane and the Hugoniot curve is represented there.

For this representation, many investigations[1] have shown (in the absence of shock-induced phase changes) that the data in the U_s–u_p plane are best represented by a straight line, i.e.,

$$U_s=C_0+Su_p. \tag{4}$$

Eliminating u_p in Eqs. 1 and 2 gives

$$U_s^2=V_0^2(P-P_0)/(V_0-V), \tag{5}$$

and the intercept C_0 in the U_s–u_p plane described by Eq. 4 is the limiting value of U_s as $P\rightarrow P_0$ and $V\rightarrow V_0$, i.e.,

$$C_0^2=-V_0^2(dP/dV)|_{P=0}. \tag{6}$$

The connection between C_0 and plane elastic wave compression is readily apparent. Consider the two types of waves that can be propagated in an isotropic solid, namely, longitudinal waves with their particle motion in the direction of the propagation and shear waves with their particle motion transverse to the direction of propagation. These waves have the respective velocities

$$C_l^2=(\lambda+2\mu)/\rho \tag{7}$$

and

$$C_s^2=\mu/\rho, \tag{8}$$

where μ and λ are the two Lamé elastic modulii and ρ is the density $(1/V)$. For a plane-wave compression of the isotropic solid, the strains

$$\epsilon_{11}=\epsilon_{22}=\epsilon_{33}=\tfrac{1}{3}\Delta V/V. \tag{9}$$

Using Eq. 9 and the fact that the modulus of bulk compressibility is the ratio of the hydrostatic pressure P to the compression $\Delta V/V$ allows the modulus of bulk compressibility to be expressed as

$$\frac{P}{\Delta V/V}=\frac{3\lambda+2\mu}{3}=\lambda+\tfrac{2}{3}\mu \tag{10}$$

and

$$P=(\lambda+\tfrac{2}{3}\mu)[(V_0-V)/V]. \tag{11}$$

Therefore,

$$C_0^2=(\lambda+\tfrac{2}{3}\mu)/\rho_0, \tag{12}$$

and the intercept for zero particle velocity is the bulk

[1] R. G. McQueen and S. P. Marsh, J. Appl. Phys. **31**, 1253–1269 (1960).

AUG 17 1970

TABLE I. Epoxy resin composition and curing condition.

Epoxy resin	Composition[a] (parts by weight)	Curing condition
Epoxy-1	50/EPON 826; 50/DER 732; 9/curing agent A	72 h at 23°C +1 h at 115°C
Epoxy-2	100/EPON 828; 14.5/curing agent CL	1 h at 115°C
Epoxy-3	70/EPON 826; 30/DER 732	18 h at 23°C +2 h at 100°C
Epoxy-4	50/EPON 826; 50/DER 732; 9/curing agent A	24 h at 23°C +2 h at 100°C

[a] Compositions are commercial designations (Shell Chemical Co.). The epoxides were polymerized with stoichiometric amounts of amine curing agents, e.g., *m*-phenylenediamine.

sound speed. Its value in terms of the Lamé constants is obtained by using the relations for C_l and C_s:

$$C_0^2 = \frac{\lambda+\frac{2}{3}\mu}{\rho_0} = \frac{\lambda+2\mu}{\rho_0} - \frac{4}{3}\frac{\mu}{\rho_0} \tag{13}$$

and

$$C_0^2 = C_l^2 - \tfrac{4}{3}C_s^2. \tag{14}$$

We note that the ultrasonic bulk sound speed is not measured directly by static means but is obtained from measurements of the longitudinal and shear sound velocities through Eq. 14.

From Eqs. 1, 2, and 4 the experimental dynamic pressures are given by

$$P = C_0^2(V_0 - V)/[V_0 - S(V_0 - V)]^2, \tag{15}$$

and differentiation yields the dynamic bulk modulus

$$-V\frac{dP}{dV} = -C_0^2 \frac{(V/V_0)\{1+S[1-(V/V_0)]\}}{V_0\{1-S[1-(V/V_0)]\}^3} \tag{16}$$

Thus, accurate estimates of C_0 and S from shock measurements permit a useful extension of dynamic compression data into low-pressure regions, which normally are accessible only to static techniques. This allows an inspection of the relationship between dynamic and ultrasonic determinations of the bulk modulus.

To determine S accurately requires extensive shock-wave measurements, although a value of S for a linear U_s–u_p relation can be calculated[2] from knowledge of the thermodynamic Grüneisen constant γ (evaluated at $P=0$) and the coefficient of volume expansion.

In the absence of these data, we have developed an optical technique,[3] using the attenuation of weak shock waves in water to deliver an approximate sonic pulse for obtaining the lower limit of shock-propagation data. This method has been used to estimate the bulk sound speeds and the isentropic bulk modulii of various solids, including propellants and explosives, to which conventional static or ultrasonic methods are often difficult to apply. The method also was used to examine the compressive behavior of several cross-linked polymers (Table I) based upon epoxide resins.[4] This paper reports these measurements and gives results for several metals, chemical compounds, and more common polymer materials.

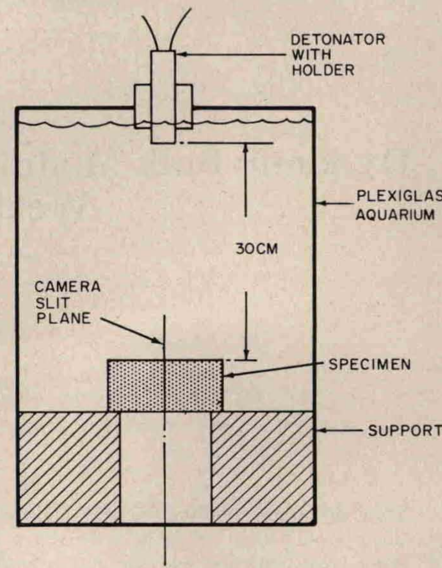

FIG. 1. Experimental assembly for initiating and measuring weak underwater shocks.

I. EXPERIMENTAL

In the arrangement shown in Fig. 1, a cylinder or slab of a specimen material is immersed in a transparent liquid, usually water contained in a Plexiglas aquarium. The specimens range in thickness from 1.27 to 5.1 cm and are positioned with their plane–parallel surfaces perpendicular to the direction of the incoming wave. An electric detonator (or exploding tungsten wire) is centered in the liquid, 30 cm from the plane surface of the specimen. The initiation of the detonator produces a very weak shock wave, which is nearly planar when it arrives, $\sim 200\,\mu$sec later, at the liquid–specimen interface.

The propagation of this incident wave normal to the specimen's parallel surface and the motion of the emerging wave transmitted by the specimen to the liquid below are recorded by a rotating-mirror smear camera along a central narrow line defined by the camera slit. The wave propagation is made visible by "back-lighting" using a shadowgraph technique.[5] In

[2] J. Berger and S. Joigneau, Compt. Rend. **249**, 2506–2508 (1959). See also D. Pastine and D. Piacesi, J. Phys. Chem. Solids, **27**, 1783–1792 (1966).

[3] J. M. Majowicz, Naval Ordnance Laboratory (unpublished rep.).

[4] These resins are Epon 828 and Epon 326 from Shell Chemical Company and are essentially diglycidyl ethers of bisphenol A, with epoxide equivalents within the range of 185 to 205. See H. Lee and K. Nevill, *Epoxy Resins* (McGraw–Hill Book Co., New York, 1957), pp. 10–11, for details of preparation and basic unit structure.

[5] T. P. Liddiard, Jr., and B. E. Drimmer, J. Soc. Motion Picture Television Eng. **70**, 106–110 (1961).